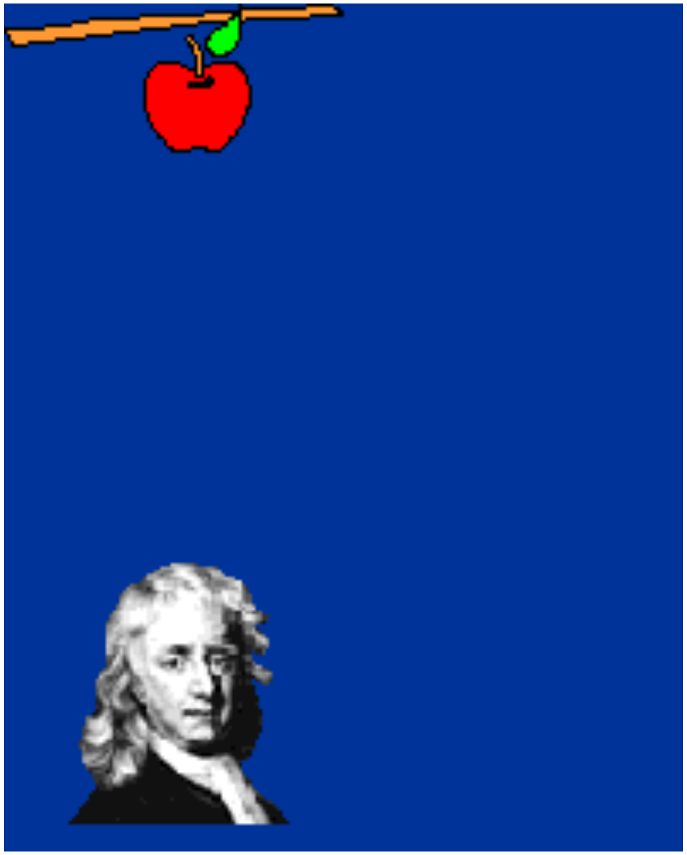
Chapter 8 - Gravity
Tuesday, March 24th

- Newton's law of gravitation
- Gravitational potential energy
- Escape velocity
- Kepler's laws
- Demonstration, *i*Clicker and example problems

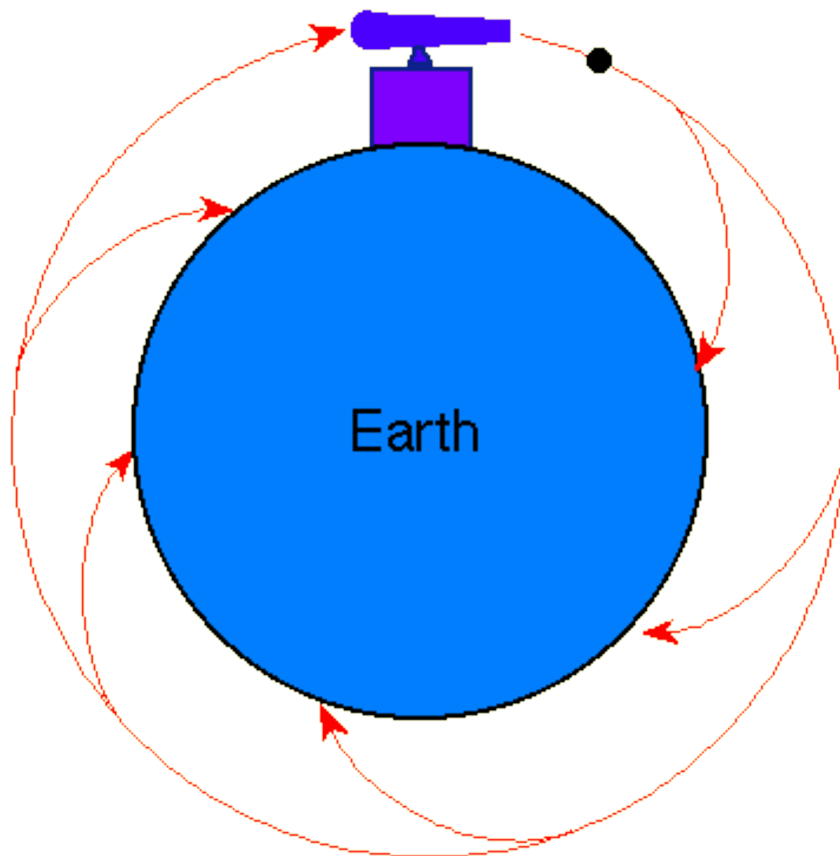
- We are jumping backwards to Ch. 8 today; Ch. 13 on Thu.
- Mini-Exam IV on Thursday (LONCAPA #13-17).

Reading: pages 118 to 129 in text book (Chapter 8)

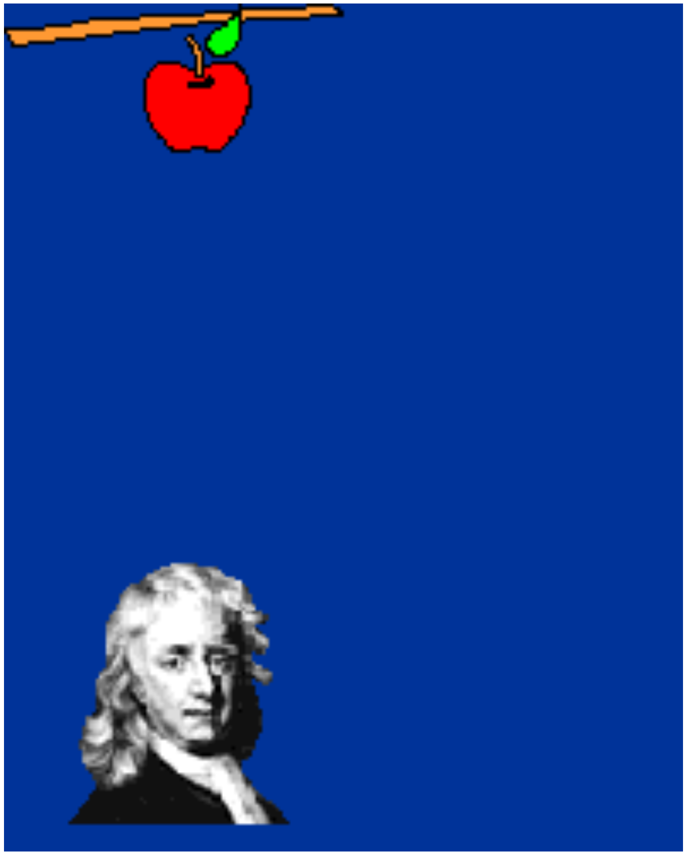
Newton's law of gravitation



- When only 23 years old, in 1665, Sir Isaac Newton showed that the force that holds the Moon in its orbit around the Earth is the same force that makes an apple fall.

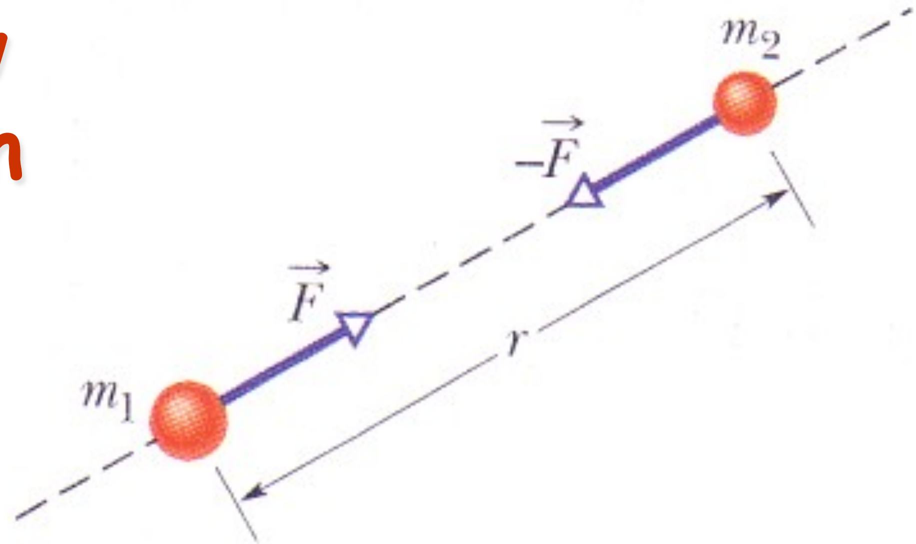


Newton's law of gravitation



- When only 23 years old, in 1665, Sir Isaac Newton showed that the force that holds the Moon in its orbit around the Earth is the same force that makes an apple fall.
- In fact, not only does the earth pull on the moon and the apple – every body in the universe attracts every other body.
- This universal tendency for masses to attract is called **gravitation.**
- Here on Earth, the earth's pull tends to overwhelm all other gravitational interactions, *e.g.*, your pull on an apple.

Newton's law of gravitation



Newton proposed a force law that we call Newton's law of gravitation:

$$F = G\frac{m_1 m_2}{r^2} \qquad G = 6.67\times 10^{-11}\,\mathrm{N\cdot m^2/kg^2}$$

- Here, m_1 and m_2 are the masses of the particles, r is the distance between them, and G is the **Gravitational constant.**
- The gravitational force of m_1 on m_2 is directed towards m_1.
- Conversely, the force of m_2 on m_1 is directed towards m_2.
- Therefore:

$$\vec{F}_1 = -\vec{F}_2$$

- Therefore, F_1 and F_2 form a third law force pair.

Gravity at the surface of the Earth

- For a sphere, e.g., the Earth, the radius that matters is the distance from the center (Shell theorem - PHY2049)

$$F_m = G\frac{mM}{r^2} = m\left(\frac{GM}{r^2}\right) = mg$$

- At the Earth's surface, $r = 6.37\times10^6$ m, and $M = 5.98\times10^{24}$ kg; thus

$$g = \left(\frac{GM}{r^2}\right) = \left(\frac{6.67\times10^{-11}\,\mathrm{N\cdot m^2\cdot kg^{-2}}\times5.98\times10^{24}\,\mathrm{kg}}{\left(6.37\times10^6\,\mathrm{m}\right)^2}\right)$$

$$= 9.83\,\mathrm{N/kg}$$

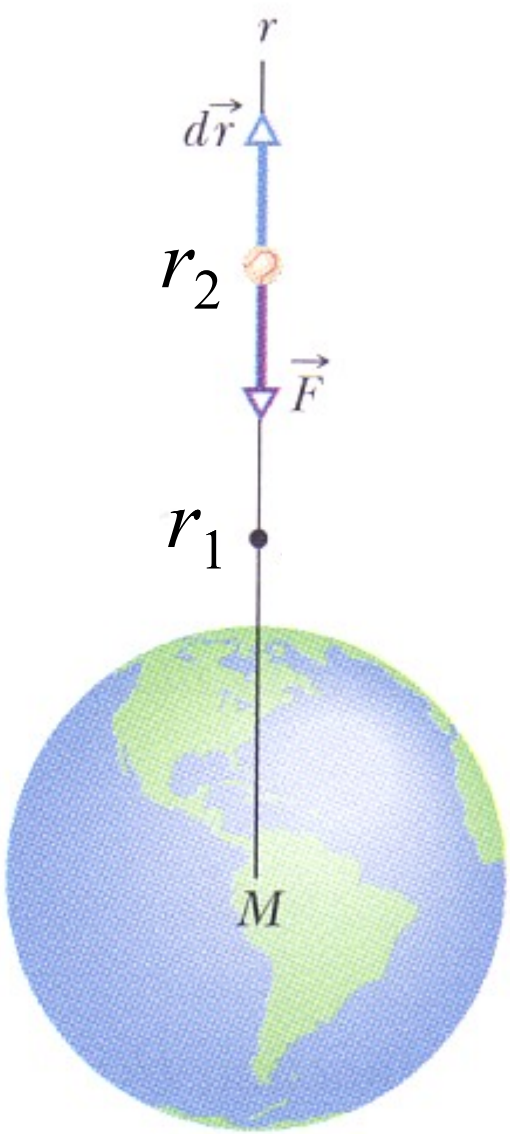
- Compared to the other fundamental forces, gravity is weak.
- Scientists have tried to unify all of the forces for centuries.
- Gravity remains the one stumbling block to grand unification.

The Earth is not flat!

- The surface of the Earth is not uniform, *e.g.* mountains.
- Furthermore, the density of the crust is not uniform.
- The surface of the Earth is not inertial (accelerating).
- Thus, g varies slightly from region to region.
- g also varies with altitude.

Altitude (km)	a_g (m/s^2)	Altitude Example
0	9.83	Mean Earth surface
8.8	9.80	Mt. Everest
36.6	9.71	Highest manned balloon
400	8.70	Space shuttle orbit
35 700	0.225	Communications satellite

Gravitational potential energy



$$\vec{F}(r) = -\frac{GMm}{r^2}\hat{r}$$

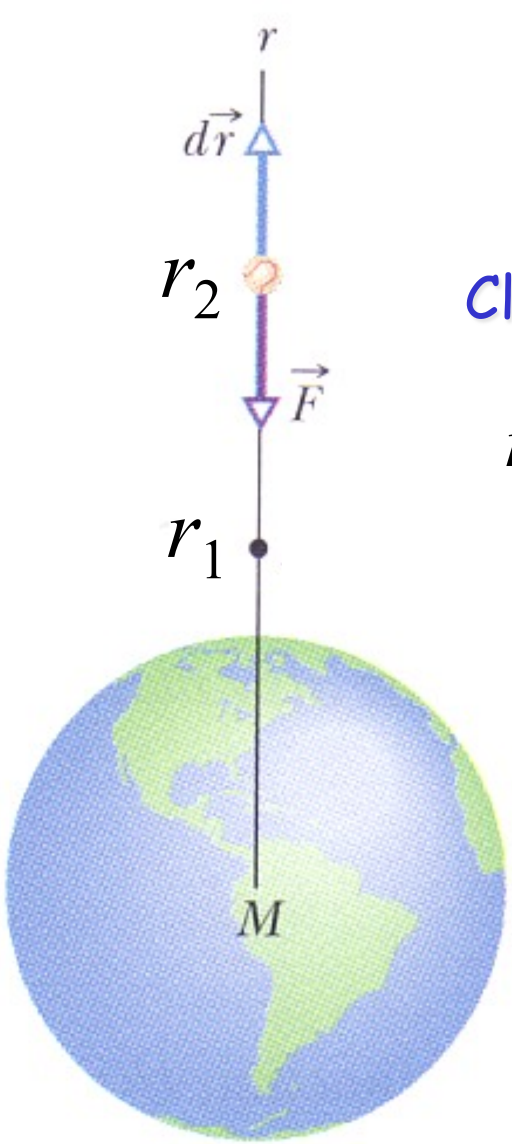
- Note, in the case of gravity, $\vec{F}$ is in the opposite direction to $d\vec{r}$ and $\hat{r}$.
- Therefore, work done by gravity:

$$W_g = \int_{r_1}^{r_2} \vec{F}(r)\cdot d\vec{r} = -\int_{r_1}^{r_2} \frac{GMm}{r^2}dr$$

$$= \frac{GMm}{r_2} - \frac{GMm}{r_1} = -\Delta U$$

- This looks rather different from mgh!!

Gravitational potential energy



$$\Delta U = \frac{GMm}{r_1} - \frac{GMm}{r_2} = m\frac{GM}{r_1 r_2}(r_2 - r_1)$$

Close to the Earth's surface:

$$r_1 \approx r_2 \approx r, \quad \Rightarrow \quad \Delta U \approx m\frac{GM}{r^2}(r_2 - r_1) = mg\Delta h$$

A universal reference point where *U* = 0

We choose this point at $r_1 = \infty$, since U must be zero there, because gravity can have no influence at infinite separation.

Then,

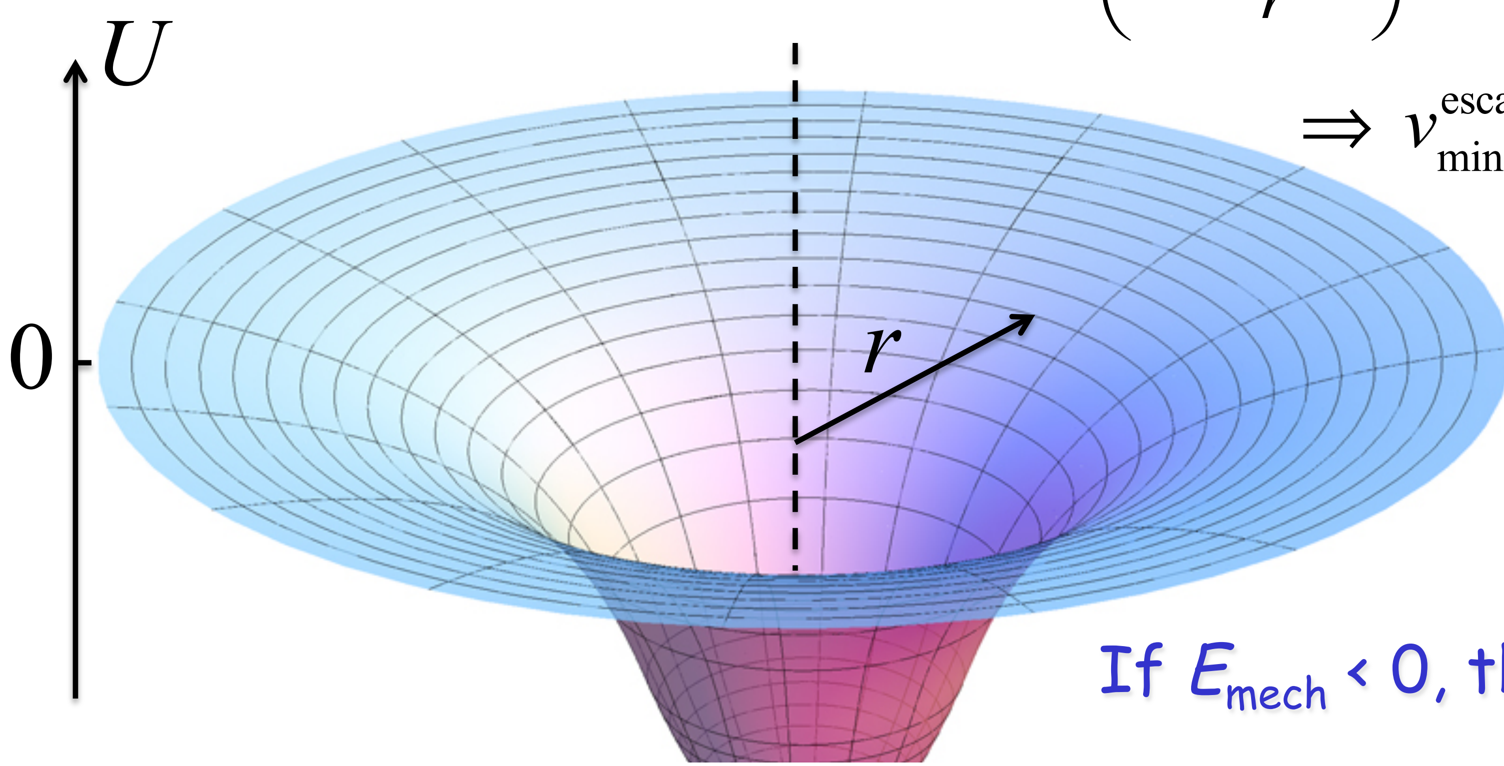
$$U(r) = -\frac{GMm}{r}$$

Escape speed

- In order to escape the gravitational pull of an object of mass M, the kinetic energy, K, must be sufficient to overcome the negative gravitational potential energy, U, i.e. the total mechanical energy must be greater than zero in order to break free of the gravitational influence of the mass.

$$E_{mech} = K + U = \tfrac{1}{2}mv^2 + \left(-\frac{GMm}{r}\right) > 0$$

$$\Rightarrow v_{\min}^{\text{escape}} = \sqrt{\frac{2GM}{r}}$$



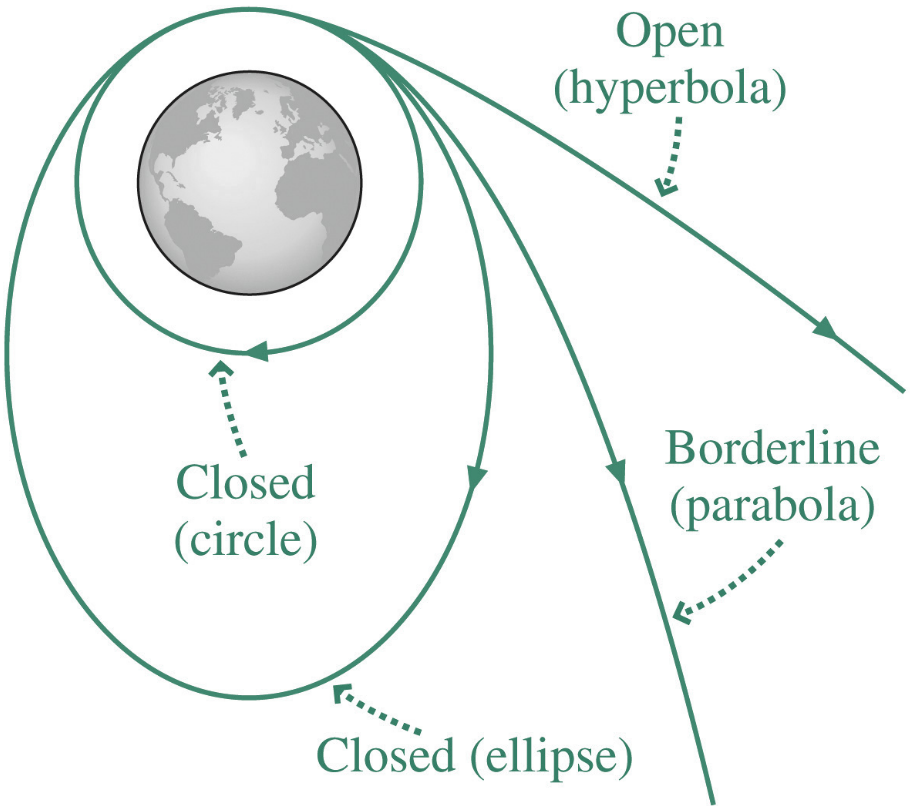
If $E_{mech} < 0$, then trapped.

Planets and satellites: Kepler's laws

- Johannes Kepler (1571 - 1630) worked out empirical laws that govern the motions of all of the planets.
- "Empirical" implies that these laws were based on observations, not on any theory.
- Kepler's laws were deduced from extensive astronomical observations made by Tycho Brahe (1546 - 1601)
- Newton (1642 - 1727) later derived Kepler's laws from his law of gravitation.

Planets and satellites: Kepler's laws

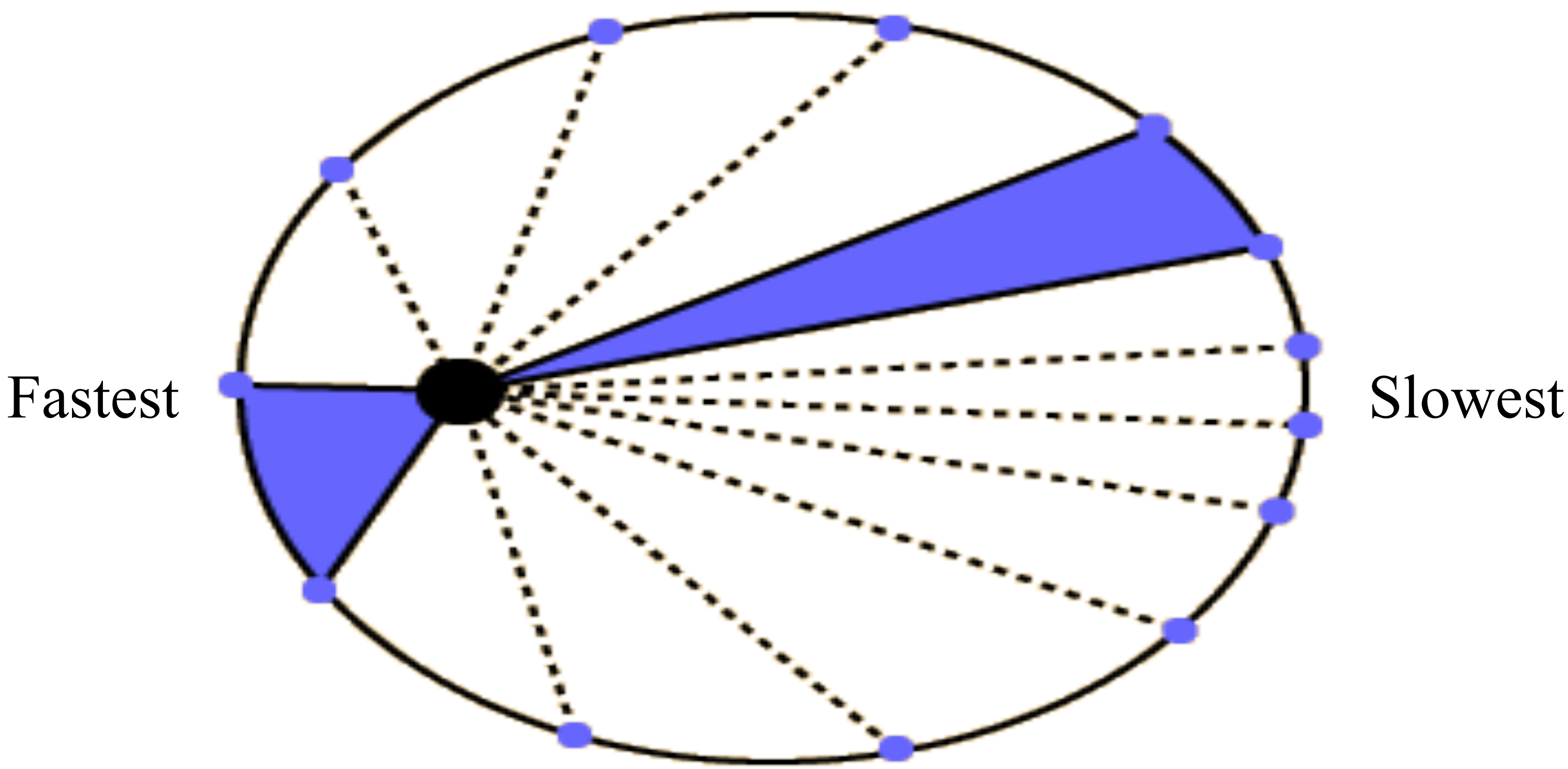
1. THE LAW OF ORBITS: All planets move in elliptical orbits, with the sun at one focus.



- $E_{mech} < 0$: The object is in a bound, elliptical orbit.
    - Special cases include circular orbits and the straight-line paths of falling objects.
- $E_{mech} > 0$: The orbit is unbound and hyperbolic.
- $E_{mech} = 0$: The borderline case gives a parabolic orbit.

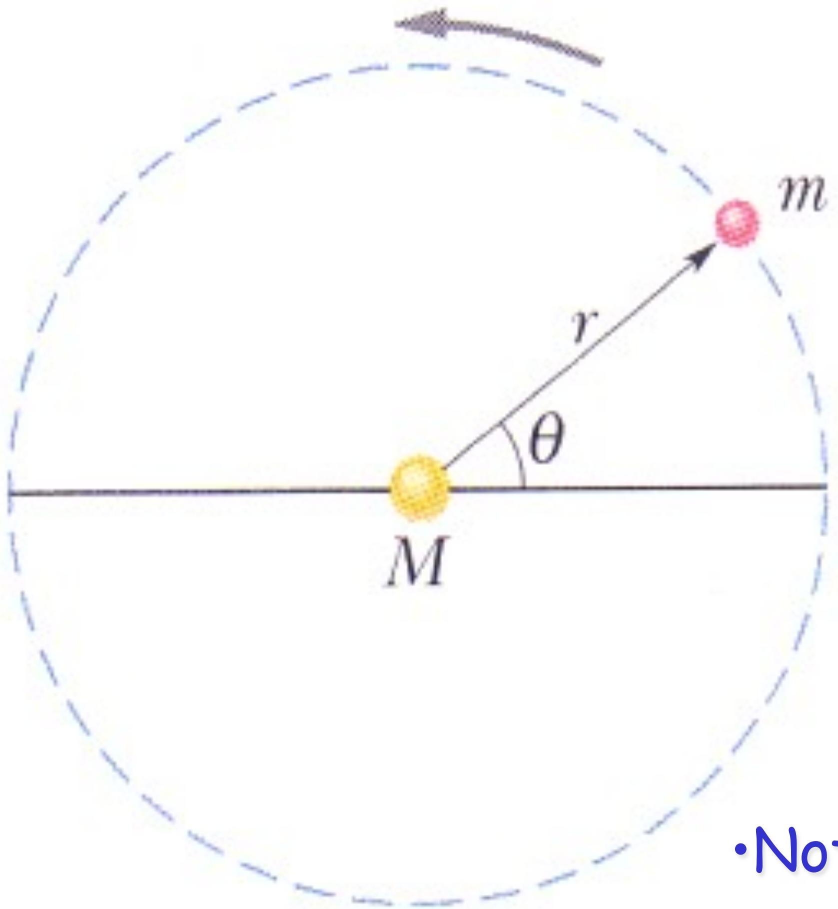
Planets and satellites: Kepler's laws

2. THE LAW OF AREAS: A line that connects a planet to the sun sweeps out equal areas in the plane of the planet's orbit in equal times; that is, the rate dA/dt at which it sweeps out area A is constant.



Planets and satellites: Kepler's laws

3. THE LAW OF PERIODS: The square of the period of any planet is proportional to the cube of the semimajor axis of the orbit.



- We'll do the math for a circular orbit, but it holds quite generally for all elliptical orbits.
- Applying *F = ma*:

$$\frac{GMm}{r^2} = (m)(\omega^2 r)$$

- Period, $T = 2\pi / \omega$

$$\Rightarrow T^2 = \frac{(2\pi)^2}{\omega^2} = \frac{(2\pi)^2}{GM} r^3$$

- Note: T is independent of m.
- Thus, $T^2/r^3 = \text{constant}$